CANDIDATE CODE _ _ _ _ _

Problem 1. Product of solutions for equation $4x^2 - 52x = 64$ is:
a) 16 b) -16 c) 13 d) -13

Solution:
This equation is equivalent to:
$4x^2 - 52x - 64 = 0.$
Viete's formula for product of solutions of quadratic equation in form
$ax^2 + bx + c = 0$,
where a, b and c are real constants is $x_1 \cdot x_2 = \frac{c}{a}$.
Constants a and c are $a = 4$ and $c = -64$, for given equation, thus the product of solutions is equal to $\frac{c}{a} = -16$. Correct answer is b.

Problem 2. The rest of division of polynomial $P(x) = 2x^5 + 2x^4 + x^2 - 12x + 1$ with polynomial $Q(x) = x + 1$ is:
a) -6 b) 14 c) 16 d) 0

Solution:
According to Bezu theorem the rest of division of polynomial $P(x)$ with polynomial $(x - a)$ is equal to $P(a)$. Applying that theorem to given polynomial $P(x)$ and $Q(x) = x - (-1)$ we can calculate the rest as:
$P(-1) = 2 \cdot (-1)^5 + 2 \cdot (-1)^4 + (-1)^2 - 12 \cdot (-1) + 1 = -2 + 2 + 1 + 12 + 1 = 14$.
Correct answer is b.

Problem 3. All values of real parameter k for which the line $p: y = kx + 2$ is a tangent line to circular $K: x^2 + y^2 = 1$ are:

a) 1 b) $\pm\frac{\sqrt{3}}{2}$ c) ± 2 d) $\pm\sqrt{3}$

Solution:
The system of equations

$$x^2 + y^2 = 1$$
$$y = kx + 2,$$

Must have only one solution in order for line and circular to have one point in common.
We can insert the second equation into first, and get

$$x^2 + (kx+2)^2 = 1.$$

The last equation is equivalent to

$$x^2(1+k^2) + 4kx + 3 = 0.$$

The starting system will have only one solution if the last equation has only one solution, and that condition is met if the equation's discriminant is equal to zero.

$$D = 16k^2 - 12(1+k^2) = 0 \Leftrightarrow 16k^2 - 12 - 12k^2 = 0,$$
$$k^2 = 3 \Rightarrow k = \pm\sqrt{3}.$$

Correct answer is d.

Problem 4. The set that presents solution of inequality $\frac{2x-5}{x+3} \le 1$ is:

a) $[8, +\infty)$ b) $(-3, 8]$ c) $[-8, 3]$ d) $(-\infty, -3)$

Solution:
Domain of definition for inequality $\frac{2x-5}{x+3} \le 1$ is set $(-\infty, -3) \cup (-3, +\infty)$. After applying basic mathematical operations the given inequality can be written in form $\frac{2x-5-(x+3)}{x+3} \le 0, \frac{x-8}{x+3} \le 0.$

	$-\infty$ -3	8	$+\infty$
$x+3$	-	+	+
$x-8$	-	-	+
$\frac{x-8}{x+3}$	+	-	+

As we can see from the table above the solution is set $(-3, 8]$. Correct answer is b.

Problem 5. Value of expression $\left|\frac{1-z}{1+z}\right|$, for $z=2i$, is:

a) 5 b) 2 c) 1 d) $\frac{\sqrt{5}}{5}$

Solution:

The module of quotient $\left|\frac{1-z}{1+z}\right|$ is equivalent to ratio of modules, thus we can write $\left|\frac{1-z}{1+z}\right|=\frac{|1-z|}{|1+z|}=\frac{|1-2i|}{|1+2i|}=\frac{\sqrt{1+(-2)^2}}{\sqrt{1+2^2}}=1$. Correct answer is c.

Problem 6. The real part $\mathrm{Re}(z)$ of complex number $z\neq 0$ is two times larger than it's imaginary part $\mathrm{Im}(z)$. How much, in percentage, is $\mathrm{Re}(z^2)$ smaller than $\mathrm{Im}(z^2)$?

a) 20% b) 25% c) 33% d) 50%

Solution:

According to condition given in the problem, for complex number $z=x+iy$ we can write $x=2y$.

$z^2=(x+iy)^2=x^2+2xyi-y^2$

Based on previous expressions we can write $\mathrm{Re}(z^2)=x^2-y^2=\frac{3}{4}x^2$ and $\mathrm{Im}(z^2)=2xy=x^2$, or $\mathrm{Re}(z^2)=\frac{3}{4}\mathrm{Im}(z^2)$ or $\mathrm{Re}(z^2)$ is 25% smaller than $\mathrm{Im}(z^2)$. Correct answer is b.

Problem 7. Given expression $A=\left(\frac{1}{a+b}-\frac{1}{a-b}\right)\left(\frac{a^2}{b^2}-1\right)$,with conditions $a\neq\pm b\wedge b\neq 0$, is equivalent to

a) $A=-\frac{2}{b}$ b) $A=\frac{2}{b}$ c) $A=-\frac{2a}{b^2}$ d) $A=\frac{2a}{b^2}$

Solution:
Expression A is equivalent to
$A=\left(\frac{1}{a+b}-\frac{1}{a-b}\right)\left(\frac{a^2}{b^2}-1\right)=\frac{a-b-a-b}{(a+b)(a-b)}\frac{a^2-b^2}{b^2}=-\frac{2}{b}$. Correct answer is a.

Problem 8. If $\cos\alpha=\frac{3}{5}$ and $\alpha\in[0,\pi]$, then $\sin\alpha$ is equal to:

a) $\frac{4}{5}$ b) $-\frac{4}{5}$ c) $\pm\frac{4}{5}$ d) $\frac{3}{5}$

Solution:
Well known trigonometric identity is: $\sin^2\alpha+\cos^2\alpha=1$, od we can write
$\sin^2\alpha=1-\cos^2\alpha\Rightarrow\sin\alpha=\pm\sqrt{1-\cos^2\alpha}=\pm\sqrt{1-\left(\frac{3}{5}\right)^2}=\pm\sqrt{\frac{16}{25}}=\pm\frac{4}{5}$.

Since $\sin\alpha\geq 0$, for angles $\alpha\in[0,\pi]$, then the answer is $\sin\alpha=\frac{4}{5}$. Correct answer is a.

Problem 9. The solutions of logaritmic equation $\log_5^2 x - \log_5 x - 6 = 0$ are x_1 and x_2. The product of these solutions $x_1 \cdot x_2$ is:

a) 5 b) 6 c) 1 d) 25

Solution:

The domain of definition for given equation is set $(0,+\infty)$. By replacing $\log_5 x = t$ we get quadratic equation

$t^2 - t - 6 = 0 \Rightarrow t_1 = -2 \vee t_2 = 3$.

And now we can write

$\log_5 x = -2 \quad \vee \quad \log_5 x = 3$

$x_1 = \frac{1}{25} \qquad x_2 = 125$

The product of solutions is 5. Correct answer is a.

Problem 10. Equation $4^x = 2^{\frac{x+1}{x}}$ has 2 solutions, x_1 and x_2. The sum $x_1^2 + x_2^2$ is equal to:

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{9}{4}$ d) $\frac{5}{4}$

Solution:

Domain of definition for given equation is set $\mathbb{R} \setminus \{0\}$. The given equality is equivalent to:

$$2^{2x} = 2^{\frac{x+1}{x}}$$

$$2x = \frac{x+1}{x}$$

$$2x^2 - x - 1 = 0 \qquad \Rightarrow x_{1,2} = \frac{1 \pm \sqrt{1+8}}{4}$$

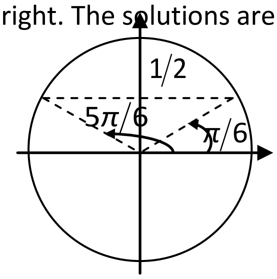
$x_1 = -\frac{1}{2}$ and $x_2 = 1$. The wanted sum is equal to $x_1^2 + x_2^2 = \frac{1}{4} + 1 = \frac{5}{4}$. Correct answer is d.

Problem 11. All solutions of trigonometric equation $2\sin\left(3x-\frac{\pi}{3}\right)=1$, that belong to segment $\left[0,\frac{\pi}{2}\right]$ are:

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ i $\frac{-\pi}{6}$ c) $\frac{\pi}{6}$ i $\frac{7\pi}{18}$ d) $\frac{\pi}{6}$ i $\frac{7\pi}{18}$ i $\frac{5\pi}{6}$

Solution:

Domain of definition for given equation is set $\mathbb{R}$. The solution of given equation can be determined if we write equation in form $\sin\left(3x-\frac{\pi}{3}\right)=\frac{1}{2}$, and look at the pisture on the right. The solutions are



$3x_1 - \frac{\pi}{3} = \frac{\pi}{6} + 2k\pi \;,\; k\in\mathbb{Z}$ $\qquad$ $3x_2 - \frac{\pi}{3} = \frac{5\pi}{6} + 2k\pi \;,\; k\in\mathbb{Z}$

$3x_1 = \frac{\pi}{2} + 2k\pi \;,\; k\in\mathbb{Z}$ $\qquad$ $3x_2 = \frac{7\pi}{6} + 2k\pi \;,\; k\in\mathbb{Z}$

$x_1 = \frac{\pi}{6} + \frac{2k\pi}{3}, k\in\mathbb{Z}$ $\qquad$ $x_2 = \frac{7\pi}{18} + \frac{2k\pi}{3}, k\in\mathbb{Z}$

For two sets of solutions, we need tosingle out those which belong to segment $\left[0,\frac{\pi}{2}\right]$ and they are:

$x_{1,0}=\frac{\pi}{6}$ and $x_{2,0}=\frac{7\pi}{18}$ (for $k=0$). Any other solution would not belong to given segment. Correct answer is c.

Problem 12. Total of 10 players applyed to chess tournament. Each player plays one match with every other player. Total number of matches played is:

a) 90 b) 60 c) 45 d) 25

Solution:

Each player plays N-1 maches with other players (if N is the number of players). Total number of maches played is $\frac{N(N-1)}{2}=\frac{10\cdot 9}{2}=45$, since we need to consider the fact that each game can not be counted twice. Correct answer is c.

Problem 13. Mark $a_{(b)}$ is a mark for number a in numeral system with base b. The sum $444_{(6)}+213_{(6)}$ is equal to:

a) $1101_{(6)}$ b) $250_{(10)}$ c) $250_{(6)}$ d) $1101_{(2)}$

Solution:

The wanted sum can directly be determined by adding in numeral system with base 6 and it is:

$$\begin{array}{r} 4\ 4\ 4 \\ +\ 2\ 1\ 3 \\ \hline 1\ {}^{1}1\ {}^{1}0\ {}^{1}1 \end{array}$$

The problem can also be solved by converting all numbers to numeral system with base 10, adding and converting the result to numeral systems with bases 2 and 6 (look at the given answers).

$$444_{(6)}+213_{(6)}=\left(4\cdot 6^2+4\cdot 6+4\right)+\left(2\cdot 6^2+1\cdot 6+3\right)=253_{(10)}$$

Converting the result to numeral system with base 6, will give the same result as direct adding in numeral system with base 6. Correct answer is a.

Problem 14. Solution of inequality $\log_2\left(x^2-3x+4\right)<\log_2 2$ is set:

a) $(1,2)$ b) $(0,2)$ c) $(-\infty,-1)\cup(2,+\infty)$ d) $(-\infty,1]\cup(2,+\infty)$

Solution:

Domain of definition for given inequality is set $\left\{x\in\mathbb{R}: x^2-3x+4>0\right\}=\mathbb{R}$. Given inequality is equivalent to:

$x^2-3x+4<2,$

$x^2-3x+2<0,$

$(x-1)\cdot(x-2)<0.$

The solution for the last, quadratic inequality is set $(1,2)$. Correct answer is a.

Problem 15. Wanted circular is concentric to circular $x^2+y^2+6x+2y+5=0$, and it crosses point T(1, -4). Equation of wanted circular is:

a) $K:(x+3)^2+(y+1)^2=25$ b) $K:(x-3)^2+(y+1)^2=16$

c) $K:(x+3)^2+(y+1)^2=16$ d) $K:(x+3)^2+(y-1)^2=25$

Solution:

Equation of given circular is:

$x^2+y^2+6x+2y+5=0 \Leftrightarrow (x+3)^2+(y+1)^2=5 \Rightarrow C(-3,-1)$, $r=\sqrt{5}$.

Equation of wanted circular is:

$K:(x+3)^2+(y+1)^2=R^2$, $T(1,-4)\in K \Rightarrow (1+3)^2+(-4+1)^2=R^2 \Rightarrow R=5$

Correct answer is a.

Problem 16. Merchandise is on 20% discount. How much does the salesman have to increase the price in order to get the starting price back?

a) 20% b) 25% c)30% d)50%

Solution:

Let's assume that starting price is x. After the discount price of the merchandise is:

$$x-20\%\cdot x = x-\frac{20}{100}x = x-0{,}2x = 0{,}8x.$$

In order to bring back the starting price we need to raise the price for $a(\%)$.

$$0{,}8x + a(\%)\cdot 0{,}8x = x,$$

$$0{,}8+\frac{a}{100}0{,}8=1,$$

$a=25.$

Correct answer is b.

Note: This problem can alo be solved if we put some number instead of x.

Problem 17. Points $D(2,3)$, $E(-1,2)$ and $F(4,5)$ are centers of sides BC, CA and AB of triangle ABC in written order. The sum of coordinater of vertex A is:

a)-2 b) -1 c) 0 d) 5

Solution:

As points $D(2,3)$, $E(-1,2)$ and $F(4,5)$ are centers of sides BC, CA and AB of triangle ABC we can write

$x_D = \frac{x_B + x_C}{2} = 2$ and $y_D = \frac{y_B + y_C}{2} = 3$,

$x_E = \frac{x_C + x_A}{2} = -1$ and $y_E = \frac{y_C + y_A}{2} = 2$,

$x_F = \frac{x_A + x_B}{2} = 4$ and $y_F = \frac{y_A + y_B}{2} = 5$,

and this is equivalent to system of equations:

$x_A + x_B = 8 \wedge x_A - x_B = -6 \Rightarrow x_A = 1$,

$y_A + y_B = 10 \wedge y_A - y_B = -2 \Rightarrow y_A = 4$,

thus the sum of coordinates of vertex A is 5. Correct answer is d.

Problem 18. Equation $(m-1)x^2 - 2mx + 3 = 0$ has real roots of opposite signs .The values of real parameter *m*,that meet the previous condition belong to interval :

a) $(-\infty,1)$ b) $(-\infty,5)$ c) $(-5,1)$ d) $(-\infty,0)$

Solution:

Quadratic equation will have real roots if discriminant *D* is $D \geq 0$. We can write for given equation $D = 4m^2 - 12m + 12$, and $D \geq 0$ for any value of real parameter *m*.

If the roots of quadratic equation are of opposite sign then the product of solutions is always less than zero $x_1 \cdot x_2 < 0 \Leftrightarrow (x_1 < 0 \wedge x_2 > 0) \vee (x_1 > 0 \wedge x_2 < 0)$.

Viete's formula for quadratic equation in form $ax^2 + bx + c = 0$, is $x_1 \cdot x_2 = \frac{c}{a} < 0$.

Comparing the last equation with given we can write

$c = 3$ and $a = (m-1)$.

Now the Viete's formula is

$\frac{3}{m-1} < 0$, $m - 1 < 0 \Rightarrow m < 1$.

For values of parameter *m* that belong to interval $(-\infty,1)$, the given quadratic equation has roots of opposite signs. Correct answer is a.

Problem 19. The set of all real solutions of inequality $\sqrt{x+3}-\sqrt{7-x}>\sqrt{2x-8}$ is:

a) $[4,7]$ b) $[4,5]\cup(6,7]$ c) $[4,5)\cup(6,7]$ d) $[4,5)\cup[6,7)$

Solution:
The domain of definition for given inequality is
$\{x\in\mathbb{R}: x+3\geq 0\wedge 7-x\geq 0\wedge 2x-8\geq 0\}=\{x\in\mathbb{R}>4\leq x\leq 7\}$.

The given inequality is equivalent to $\begin{matrix} x+3>7-x+2\sqrt{(7-x)(2x-8)}+2x-8 \\ 2>\sqrt{(7-x)(2x-8)} \end{matrix}$. As the left side is positive number, we can square the las inequality $\begin{matrix} -2x^2+22x-60<0 \\ -x^2+11x-30<0 \end{matrix}$. The solution of last inequality is set $(-\infty,5)\cup(6,+\infty)$. If we consider the domain the total solution of given inequality is set $[4,5)\cup(6,7]$. Correct answer is c.

Problem 20. x is an acute angle. The solution of inequality $\sin x+\sqrt{3}\cos x>\sqrt{3}$ is interval:

a) $\left(0,\frac{\pi}{2}\right)$ b) $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ c) $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ d) $\left(0,\frac{\pi}{3}\right)$

Solution:

The domain of given inequality is set $\left(0,\frac{\pi}{2}\right)$. The given inequality is equivalent to

$$\begin{matrix} \sin x+\sqrt{3}\cos x>\sqrt{3} \\ \sin x+\sqrt{3}\sin\left(\frac{\pi}{2}-x\right)>\sqrt{3} \end{matrix} \qquad \begin{matrix} \sin x\cdot\frac{1}{2}+\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{2}-x\right)>\frac{\sqrt{3}}{2} \\ \sin\left(x+\frac{\pi}{3}\right)>\frac{\sqrt{3}}{2} \end{matrix},$$

$$\begin{matrix} \frac{\pi}{3}+2k\pi<x+\frac{\pi}{3}<\frac{2\pi}{3}+2k\pi, k\in\mathbb{Z} \\ 2k\pi<x<\frac{\pi}{3}+2k\pi, k\in\mathbb{Z} \end{matrix}$$

, and the total solution are only acute angles or $0<x<\frac{\pi}{3}$.

Correct answer is d.